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Another look at the angular distributions of the $\gamma\gamma \to \pi\pi$ reactions.

A.E.Kaloshin[a], V.M.Persikov[a] and V.V.Serebryakov[b]

[a] *Institute of Applied Physics, Irkutsk State University, 664003 Irkutsk, Russia*

[b] *Institute for Mathematics, 630090 Novosibirsk, Russia*

Abstract.

We analyze the existing data on the angular distributions of the $\gamma\gamma \to \pi^+\pi^-,\ \pi^0\pi^0$ reactions with using of the unitary model for helicity 2 amplitude. The purpose is to obtain the D–wave parameters and S–wave cross section. We obtain from experiment in the first time the values of $\alpha + \beta$ for sum of the electric and magnetic pion's polarizabilities. We found the S-wave cross sections much smaller as compared with previous similar analysis. Comparison of the $\gamma\gamma \to \pi^+\pi^-$ and $\gamma\gamma \to \pi^0\pi^0$ data gives an indication for a marked I = 2, J = 2 contribution in region of $f_2(1270)$.

1. Introduction.

Nowadays there exist few experiments on the angular distributions of the $\gamma\gamma \to \pi\pi$ reactions: with charged pions from CELLO [1] and MARK-II [2] detectors and with neutral pions from Crystal Ball [3]. The most evident feature of these data is the resonance peak $f_2(1270)$ interfering with smooth background. But the most interesting physically is the S-wave cross section and this interest is related with long-standing problem of scalar mesons whose properties and spectrum are looking as mysterious. Note that there is another not investigated earlier aspect of low energy physics – that's a possibility of experimental study of the sum $(\alpha+\beta)^{\pi}$ of electric and magnetic pion's polarizabilities. The only experimental information on these values is the restriction for $(\alpha+\beta)^{\pi^+}$ from experiments on nuclei [4]. [1] We found that even existing $\gamma\gamma \to \pi\pi$ experiments allow to have much more exact information on $\alpha+\beta$, the main problem here is an investigation of systematical inaccuracy in such an analysis. The matter is that one needs to extrapolate formulae with obtained parameters from $s \sim 1\ GeV^2$ to point s = 0.

The question is appearing how to use the existing experimental data on angular distributions to obtain the physical information. It's not so evident question, so let's say few words on this matter.

In ideal situation of high quality data the most preferable way is the model–independent partial wave analysis. But really it's not so profitable with present data. Such an attempt was made in [6] and it was found that results are very indefinite because of incomplete solid angle ($\mid cos\Theta \mid < 0.6$) of detector and presence of higher spin waves in $\gamma\gamma \to \pi^+\pi^-$ from QED (π - exchange) mechanism.

The second possibility, realized in [1], consists in using of the model for dominating helicity 2 amplitude T_{+-}, where the main physical effects are more or less clear. This way allowed to obtain from angular distributions the most interesting S-wave cross section with low statistical errors. Evidently this way needs the accurate modelling of the main contribution.

The third way [7] consists in using of the model expressions for both helicity amplitudes T_{+-} and T_{++}. The main problem here is related with the modelling of I = 0 S-wave and is generated by the contradictory situation and insufficient understanding of

[1] Let's recall that the sum $\alpha+\beta$ plays the role of the D-wave threshold structure constant in helicity 2 amplitude, similar to electromagnetic radius in a formfactor. As for the S-wave parameter $\alpha-\beta$, the existing near–threshold data $\gamma\gamma \to \pi\pi$ allow to get the first information on it, see [5].

hadron dynamics in this sector. In this case one should use the multi–channel approach and it needs the additional model assumptions with an additional uncertainty.

Our starting point is the desire to look at the S-wave contribution in the maximal model–independent way. So we choose the second way a' la CELLO with modeling of the main contribution and in the following we have all pluses and minuses of this choice. As compared with [1] we use another (more developed) model for dominating amplitude and include into consideration also the $\gamma\gamma \to \pi^0\pi^0$ reaction.

2. Model for helicity 2 amplitude.

We shall use the model [8] for helicity 2 amplitude which has the following properties:

1. The lowest wave with J = 2, I = 0 satisfies the one–channel unitary condition. As it was shown e.g. in [9], the inelastic effects in the process $\gamma\gamma \to f_2(1270) \to \pi\pi$ are rather small.
2. The J = 2, I = 2 wave does not take into account the final state interaction effects and contains only Born contributions.
3. The model satisfies the low energy theorem [10] not only on the level of Tompson limit but with accounting of first structural correction too.
4. It satisfies also the unsubtructed dispersion relation at fixed t [11].

Helicity amplitudes in CMS are defined in a standard manner. The cross section $\gamma\gamma \to \pi^+\pi^-$ is (for neutral pions there appears an additional factor 1/2!):

$$\frac{d\sigma}{dcos\Theta} = \frac{\rho(s)}{64\pi s} \left\{ \mid T_{++} \mid^2 + \mid T_{+-} \mid^2 \right\} \tag{1}$$

Here $\rho(s) = (1 - 4\mu^2/s)^{1/2}$, $\mu = m_\pi$. Let's pass over to reduced helicity amplitudes M which are free of kinematical singularities and zeroes [11].

$$T_{++} = sM_{++}, \qquad T_{+-} = (tu - \mu^4)M_{+-} \tag{2}$$

The reduced helicity 2 amplitudes [8] with minimal modification are looking as :

$$\begin{aligned} M^C_{+-} &= \frac{1}{\sqrt{3}D_f(s)} \left[C^0 + H^0_V(s) \right] + W^C(s,t) \\ M^N_{+-} &= \frac{1}{\sqrt{3}D_f(s)} \left[C^0 + H^0_V(s) \right] + W^N(s,t) \end{aligned} \tag{3}$$

Here and below C means "Charged" and N "Neutral".
The background contributions have the form:

$$W^C(s,t) = W^\pi(s,t) + \sum_{V=\rho,b_1,a_1} Z^V \left(\frac{1}{m_V^2 - t} + \frac{1}{m_V^2 - u} - \frac{1}{m_V^2}\right) - 5\frac{Z^{a_2}}{m_{a_2}^2 + s} + a^C \frac{s_{eff}}{s + s_{eff}}$$

$$W^N(s,t) = \sum_{V=\rho,\omega,h_1,b_1} Z^V \left(\frac{1}{m_V^2 - t} + \frac{1}{m_V^2 - u} - \frac{1}{m_V^2}\right) + a^N \frac{s_{eff}}{s + s_{eff}} \tag{4}$$

Here $W^\pi(s,t)$ is the contribution of π–exchange (QED), $Z^R = g_{R\pi\gamma}^2/4$. Besides the known resonance exchanges we allow the presence of contributions which are not taken into account exactly – these are the terms proportional to arbitrary constants a^C, a^N. Firstly, the existing experimental information on decays $R \to \pi\gamma$ allows the marked freedom in the cross– contributions. Secondly, the introduced terms should absorb in a some way and another physical effect: the modification of π–exchange due to presence of some off-shell formfactor. The existence of parameter s_{eff} accounts additional arbitrariness in extrapolation to point $s = 0$.

$D_f^{-1}(s)$ is the propagator of $f_2(1270)$–meson with accounting of the finite width corrections.

$$D_f(s) = (m_f^2 - s)\cdot(1 + ReH'(m_f^2)) - ReH(s) + ReH(m_f^2) - im_f\Gamma_f(s) \tag{5}$$

$\Gamma_f(s)$ is chosen in a standard way with accounting of centrifugal barrier.

$$\Gamma_f(s) = \Gamma_f \cdot \left[\frac{q_\pi(s)}{q_\pi(m_f^2)}\right]^5 \cdot \frac{D_2(rq_\pi(m_f^2)}{D_2(rq_\pi(s)}, \qquad D_2(x) = 9 + 3x^2 + x^4 \tag{6}$$

The parameter r defines the D–wave scattering length a_2^0.

Function $H_V^0(s)$ in (3) is the rescattering contribution.

$$H_V^0(s) = \frac{s}{\pi}\int \frac{ds'}{s'(s'-s)} m_f\Gamma_f(s')V^0(s') \ , \tag{7}$$

where $V^0(s)$ is projection of background contributions (4) onto J = 0, I = 0 state. The function $R(s) = C^0 + Re\ H_V^0(s)$ at resonance point defines the two–photon decay width $f_2 \to \gamma\gamma$. [2]

$$R(m_f^2) = \frac{\sqrt{3}\cdot 2^5 \cdot 5\pi}{m_f^3}\sqrt{\Gamma_f \cdot \Gamma(f \to \gamma\gamma)\cdot BR(f \to \pi\pi)} \tag{8}$$

[2] In real case of the marked background, interfering with $f_2(1270)$, the question what we should call by the two–photon width becomes transparently unambiguous at current experimental accuracy. The considerable deviation of values $f_2 \to \gamma\gamma$ from different groups data may be explained in part by different definitions.

The pion polarizabilities are defined at the point s = 0 (e.g. [8]) and we have from (3) :

$$\begin{aligned}
\frac{(\alpha+\beta)^C}{2\mu} &= \frac{C^0}{\sqrt{3}D_f(0)} + \sum_{V=\rho,b_1,a_1} \frac{Z^V}{m_V^2} - 5\frac{Z^{a_2}}{m_{a_2}^2} + a^C \\
\frac{(\alpha+\beta)^N}{2\mu} &= \frac{C^0}{\sqrt{3}D_f(0)} + \sum_{V=\rho,\omega,h_1,b_1} \frac{Z^V}{m_V^2} + a^N
\end{aligned} \tag{9}$$

The amplitudes (3) contain three essential parameters C^0, a^C, a^N. Instead of them we can use another set of parameters (linear combinations of the first set) $R^0 = C^0 + Re\ H_V^0(m_f^2),\ (\alpha+\beta)^C,\ (\alpha+\beta)^N$, which is preferable from our point of view. Passing to this set is rather simple.

3. Discussion of the CELLO analysis.

As it was said in Introduction, we shall analyze an angular distributions in the same semi–model manner [1], which is looking as the most attractive. For helicity 2 amplitude we use the model expression with few free parameters but the S–wave contribution is extracted independently in every energy bin. So it's necessary to say few words about the results of [1] where the data of [1] and [2] $\gamma\gamma \to \pi^+\pi^-$ were analyzed. They used for helicity 2 amplitude a rather simplified expression, satisfying the one–channel unitary condition. In symbolic form:

$$T_{+-} = T_{+-}^{QED} + B. - W.\ f_2(1270) \tag{10}$$

But if to look on results [1] more carefully, there are rather unexpected statements.

1. CELLO and MARK–II data need the significant damping of the QED contribution, in 1 GeV region the value of this damping is few times in any case.

2. There was found the S–wave contribution of unexpectedly big scale: $\sigma^S(\gamma\gamma \to \pi^+\pi^-, \mid cos\Theta \mid < 0.6) \sim 60 - 80$ nbarn at $\sqrt{s} = 0.8 - 1.0$ GeV. It differs significantly from natural scale ~ 10 nbarn, appearing both at the simple estimates [8], based on polarizability, and at the extrapolation of near–threshold analysis [5] to this region.

It turns out (see for more details [12]) that these results are based on the specific assumption about form of the Breit–Wigner contribution in (10). It was taken as:

$$\frac{d\sigma^{BW}}{d \mid cos\Theta \mid} = 40\pi \cdot \frac{m_f^2}{s} \cdot \frac{\Gamma_{\gamma\gamma}\ \Gamma_f(s)\ BR(f \to \pi^+\pi^-)}{\mid m_f^2 - s - im_f\Gamma_f(s) \mid^2} \cdot \mid Y_{22}(cos\Theta) \mid^2 \tag{11}$$

The expression (11) is valid for narrow resonance, when the chosen s–dependence in nominator is unessential, but for rather broad f_2 such choice is looking as arbitrary. This assumption is very essential for analysis [1], so let's look at consequence of (11). $\Gamma_f(s)$ was chosen as : [3]

$$\Gamma_f(s) = \Gamma_f \cdot \left[\frac{q(s)}{q(m_f^2)} \right]^5 \cdot \frac{m_f}{\sqrt{s}} \cdot f^2(s) \tag{12}$$

f(s) is so called factor of centrifugal barrier. Returning from (11) to the lowest partial wave one can find :

$$(M_{+-})^{J=2,I=0} = R(s)/(m_f^2 - s - im_f\Gamma_f(s)), \qquad R(s) = \frac{const}{s}\, f(s) \tag{13}$$

The main difference between the model (3) and [1] consists in behavior of R(s). In the model [8] the effective "coupling constant" $R(s) = C^0 + Re\ H_V^0(s)$ is defined by rescattering effect and is much more smooth function in vicinity of resonance. Moreover, the appearance of the pole in R(s) (13) breaks the low energy theorem requirements. In an analysis the value $R(m_f^2)$ is fixed well by data. However in case of (13) the function R(s) grows essentially below the resonance and it gives too big D–wave cross section exceeding the experiment. Just this fact leads to necessity to damp the QED contribution and in the end gives all above mentioned results. The repetition of analysis with another model [8] gives other results:

a) For description of experimental data both on total cross section and angular distributions it's not necessary any additional QED damping at the same or even better quality of description.

b) The extracted S–wave contribution $\sigma^S(|\ cos\Theta\ | < 0.6)$ is much less than was it found in CELLO analysis and does not contradict to results of threshold analysis [5].

4. Analysis of angular distributions.

Our helicity 2 amplitudes (3) contain three parameters: $R^0 = C^0 + Re\ H_V^0(m_f^2)$, $(\alpha+\beta)^C$, $(\alpha+\beta)^N$. We found at numerical investigation, that corresponding cross section of $\gamma\gamma \to \pi^+\pi^-$ depends very weakly on the $(\alpha+\beta)^N$ and $\gamma\gamma \to \pi^0\pi^0$ practically does not depend on $(\alpha+\beta)^C$. So for a single reaction we can use the two–parameter

[3]In fact in [1] it was put f(s)=1, the introduction of any decreasing with s factor makes all the problems even stronger.

expression fixing the alien polarizability somewhere in theoretically expected region. One can change it in a few times without any marked changing in results. Recall that different low energy quantum field models [13, 14] give rather close values : $(\alpha+\beta)^{\pi^+} \simeq 0.20$, $(\alpha+\beta)^{\pi^0} \simeq 1.20$ in units of $10^{-42}\ cm^3$. [4] Some greater values are predicted by the dispersion sum rules [15] and two–loop calculations chiral model calculations [16] (see Table 2 for more details).

So we shall describe the experimental angular distributions in the following way:

$$\frac{d\sigma}{dcos\Theta} = a_S \ + \ \frac{\rho(s)}{64\pi s}\ (tu-\mu^4)^2 \cdot \mid M_{+-}(s,t) \mid^2 \tag{14}$$

and use the model (3) with two free parameter for helicity 2 amplitude for every reaction.

We found also another source of uncertainty related with the parameter r in centrifugal barrier (6) or in other words with D–wave scattering length. As for this parameter, it practically does not change the χ^2 value in analysis influencing ,however, on the extracted low energy parameters.

So the sources of systematical inaccuracy in our analysis are the uncertainties in parameter r (D-wave scattering length) and in model for background (4) interfering with resonance.

Let us restrict ourselves in analysis by the region $E < 1.5\ GeV$, and take m_{f_2} , Γ_{f_2} from PDG Tables [17]. At the first step let's fix the value r in (6) by $r = 5.5\ GeV^{-1}$, it corresponds to the standard scattering length value $a_2^0 = 1.7 \cdot 10^{-3}$ in units of pion mass. This scattering length was obtained from experiment [18] $a_2^0 = (1.7 \pm 0.3) \cdot 10^{-3}$ and it was used to fix counterterms in the chiral model loop calculations [19, 16].

The results of our analysis at fixed $r = 5.5\ GeV^{-1}$ are shown in Table 1. There are indicated few variants corresponding to different forms of background contribution. Let us note few facts seen from this Table.

- The quality of description in all cases is satisfactory with exception ,may be, the CELLO data. In this case , however, quality of description is better than in analysis [1] of the same data. Indeed $\chi^2/NDF = 81.4/53$ in [1] and 69/51 in our analysis.

- In all variants of description the sum $\alpha+\beta$ is defined with very small statistical

[4] We use the units system $e^2 = 4\pi\alpha$, where the values of polarizabilities differ by factor 4π as compared with the system $e^2 = \alpha$. Below we shall use the units $10^{-42}\ cm^3$ for polarizabilities not indicating them.

error. [5] It means that $d\sigma/dc$ in vicinity of $f_2(1270)$ (D–wave parameters are defined in main by resonance vicinity due to big statistical weight) is extremely sensitive to value of background contribution.

- Both parameters $\alpha+\beta$ for π^+ and π^0 are lying in expected regions.
- The S–wave contributions near 1 GeV are much less than in [1] in all cases and correspond on the scale to results of near–threshold analysis [5] and to old estimates [8] based on the value $\alpha-\beta$. The typical scale for the obtained S-wave cross section is $\sim$ 10 nbarn.

Our results for polarizabilities are summarized in Table 2 in comparison with existing predictions for these values. Recall that it was obtained with standard parameters of $f_2(1270)$, generally accepted form of $\Gamma_f(s)$ and standard scattering length.

<u>On the S–wave contributions.</u>

Together with D–wave parameters, shown in Table 1, we obtain the S–wave contributions in every energy bin (14). Our results for them, corresponding to variant 2 of Table 1, are shown in Figures 1–3. The other variants of Table 1 have qualitatively the same behavior. Note that we allow parameter a_S in (14) to be negative in a fit. As a result we see that the S–wave contributions turn out much less than in analysis [1]. In case of CELLO data one can see some indications on the scalar meson $\epsilon(1300)$ production in this process but we can see from Fig. 1–3 that resonance picture is not so transparent. For numerical estimate let's consider the CELLO S–wave, assuming the resonance production with mass 1200 MeV and width 300 MeV. Then the extracted cross section of Fig. 1 corresponds to $\Gamma(\epsilon\to\gamma\gamma)\cdot BR(\epsilon\to\pi\pi)\sim 3.6\ KeV$ –see the curve. For MARK-II and Crystal Ball data there is no evident resonance–like picture and S-wave is less than in CELLO case.

One can see from these Figures one exclusive case: that's for MARK–II data in the region $E\le 0.9\ GeV$, where the S-wave cross section is formally negative. It happens in all variants 1–4 of Table 1. This circumstance practically does not influence on the D–wave parameters since they are defined mainly by resonance region. Besides, this "negative cross sections" have rather small value as compared with total cross sections.

It's not so difficult to understand the origin of this effect. Sure the cross section

[5] Recall that the only experimental information on the sum $\alpha+\beta$ is the following: $(\alpha+\beta)^C = 1.8\pm3.9\pm3.1$ [4].

$\gamma\gamma \to \pi^+\pi^-$ differs from QED one because of final state interaction effects. But at the standard form of $\pi\pi$–interaction (i.e. smooth extrapolation of $\pi\pi$ phase shift from resonance to threshold with the positive scattering length) the D–wave cross section exceeds the QED one in this region. [6] However in the region $E \leq 0.9\ GeV$ the MARK–II data in contrast to CELLO are below then the QED contribution – see Fig.4 . Naturally with given type of analysis (model for helicity 2 amplitude) there is no place for S–wave and these contributions will be negative. Let us note that and previous experiments $\gamma\gamma \to \pi^+\pi^-$ (measuring of integral cross section) differ from each other in this aspect: some of them obtain the cross section higher than QED curve, and some lower.

On two–photon width of $f_2(1270)$.

At more detailed looking at Table 1 one can see the disagreement in two–photon coupling constant R^0 between $\gamma\gamma \to \pi^+\pi^-$ and $\gamma\gamma \to \pi^0\pi^0$ experiments. To demonstrate it let us list the two–photon width, corresponding to the variant 2 of Table 1, with statistical errors only (systematical ones are much less). [7]

$$\begin{aligned} CELLO: &\quad \Gamma(f_2 \to \gamma\gamma) = 2.95 \pm 0.13\ KeV \\ MARK-II: &\quad \Gamma(f_2 \to \gamma\gamma) = 2.84 \pm 0.18\ KeV \\ CrystalBall: &\quad \Gamma(f_2 \to \gamma\gamma) = 3.70 \pm 0.22\ KeV \end{aligned} \tag{15}$$

Even taking into account that this discrepancy is related with different experiments, we see that the difference may reach to three standard deviations and it should be considered seriously. So let's consider the possible physical reasons for it. [8]

- First of all this suggests that one should take into account the effects of final state interaction in J = 2, I = 2 wave too. There exist some experimental information on this phase shift δ_2^2 : it is slow and negative in wide region (see, i.g.[20]). Indeed, we made such an attempt and found that this effect reduces the difference. But

[6] Introducing of some formfactor to QED vertex does not help here if you do not break the low energy theorem. Besides, this degree of freedom is absorbed rather well by our "effective cross–exchange" . The same problem but in much more sharp form was ,evidently, in analysis [1], which has been lead to necessity of additional "damping factor" breaking the low energy theorem at the level of structure corrections.

[7] Here we shall accept $\Gamma(f_2 \to \gamma\gamma) \sim (R^0)^2$ for simplicity. In fact we would not like to discuss in this paper what is most correct definition for decay width in this case.

[8] We mentioned in above that the D–wave parameters are defined mainly by the resonance vicinity only. So we think this discrepancy is not related with problem of negative S–wave, if it really exists.

its influence is too small: roughly speaking we shall have the difference about two standard deviations instead of three.

- Perhaps the data indicate on deviation of the $f_2(1270)$ parameters from generally accepted. Considering the mass and total width as free parameters we found the best χ^2 at $m_{f_2} = 1.28$ GeV and $\Gamma_{f_2} = 230$ MeV. But it does not reduce the difference in R^0. Besides, the problem of negative S–wave contributions becomes much more sharp. So this possibility seems to be unreasonable.

- One more possible reason: if in the π–exchange the above mentioned off–shell formfactor plays the essential role. In the lowest partial wave this effect is absorbed by our "effective cross–exchange" (we checked it in few examples) but the higher spin waves $J > 2$ in (3) do not contain this effect. As a result of corresponding calculations we came to conclusion that this effect works in the opposite direction: any damping of higher spin waves $J > 2$ leads to stronger contradiction for the two–photon coupling constant R^0.

We came to opinion that most probable reason of this disagreement in $f_2(1270)$ $\gamma\gamma$ coupling is related with some non–standard D–wave dynamics with I = 2. Let's recall that the observed in the processes $\gamma\gamma \to \rho\rho$ anomaly near threshold ($\sigma(\gamma\gamma \to \rho^0\rho^0) \gg \sigma(\gamma\gamma \to \rho^+\rho^-)$) is interpreted almost unambiguously as an exotic resonance I = 2 production (see e.g. discussion in [21]). But this effect can not be considered in framework of one–channel approach and is far away of purposes of present work.

Finally, what will be changed in results if to vary the parameter r in the centrifugal barrier ? Let it changes in interval $4.0 < r < 6.0\ GeV^{-1}$, it corresponds to D–wave scattering length between $0.6 \cdot 10^{-3}$ and $2.2 \cdot 10^{-3}$. We shall have for polarizabilities :

$$\begin{aligned} CELLO: &\quad (\alpha+\beta)^C = 0.37 \pm 0.08(stat.) \pm 0.10(syst.) \\ MARK-II: &\quad (\alpha+\beta)^C = 0.23 \pm 0.09(stat.) \pm 0.12(syst.) \\ CrystalBall: &\quad (\alpha+\beta)^N = 1.40 \pm 0.10(stat.) \pm 0.26(syst.) \end{aligned} \tag{16}$$

As for two–photon coupling constant, it is very stable and the S–wave contributions will have practically the same behavior.

5. Conclusions.

Thus, we performed the semi–model analysis a' la CELLO [1] of existing data on the angular distributions of $\gamma\gamma \to \pi\pi$ for both reactions. In contrast to [1] we used another model for the helicity 2 amplitude [8] which does not break the low energy theorem requirements.

We came to natural conclusion that such kind of analysis needs the very accurate modelling of dominating amplitude. The essential difference between our results and [1] tells that one should utilize in the model an information on $\pi\pi$ - interaction in rather wide region. The control of threshold parameters both of hadron and electromagnetic amplitudes is very useful here.

Another our observation: the angular distributions in resonance vicinity are very sensitive (especially for neutral pions) to background value. This degree of freedom is absolutely necessary for data describing. We gave a physical sense to these degrees of freedom, relating them with pion's polarizabilities, but it's not a necessary step.

There are few facts which are convinced ourselves in correctness of our approach:

- In our analysis the S–wave cross sections in region of 1 GeV and below have the typical scale about 10 nbarn, which corresponds to reasonable values of difference of polarizabilities [5, 8].

- There is no necessity for introducing of any additional damping of QED contributions at least in the first approximation. Even if here is a problem, it is much more soft as compared with [1].

- The obtained values for polarizabilities sum both for π^+ and π^0 in any variant do not contradict to theoretical predictions (see Table 2).

We came to conclusion that using of the model [8] for helicity 2 amplitude leads to rather agreed picture at least on the level of large contributions. We met some contradictions in our analysis too, but so to speak on the next level. The contradictions appear either with rather small S–wave contributions or at comparison of different experiments. The most serious one is the difference in the two–photon coupling constant of $f_2(1270)$ from $\gamma\gamma \to \pi^+\pi^-$ and $\gamma\gamma \to \pi^0\pi^0$ experiments. In our opinion it tells about new physical effect not included into standard description.

As for "negative" cross sections in MARK-II data (see Fig.2): the appearance of this effect is related with chosen form of analysis. But we think that here exists also the pure experimental problem of more exact calibration of the measured cross section – see Fig.

4 for illustration. We have in mind the location of experimental cross section relatively the π–exchange contribution's curve – sure that's much more delicate question than the cross section measuring.

We suppose that the physical results of the performed analysis are the following:

- We obtained from two–photon experiments in the first time the sum of polarizabilities both for π^+ and π^0. The existing data allow to extract the background contributions interfering with resonance $f_2(1270)$ with very small statistical errors. Thought there exists some freedom at the extrapolation to point s = 0, it is not so big as one could think from the beginning. It's surprising, but due to existing of the "amplifier" $f_2(1270)$, there are even better conditions for obtaining the D–wave parameter $\alpha + \beta$ from data than for the S–wave one $\alpha - \beta$.

- The obtained S–wave cross sections are rather small parts of the total cross sections, their scale is about 10 nbarn. There exists some resonance–like enhancement near 1.3 GeV of rather small amplitude in CELLO case. The obtained S–wave in region of $E \leq 1$ does not contradict to results of near–threshold analysis [5].

- We observe the statistically meaningful difference between $\gamma\gamma \to \pi^+\pi^-$ and $\gamma\gamma \to \pi^0\pi^0$ experiments in value of $f_2(1270)\gamma\gamma$ coupling. The most probable reason is the existing of non–standard dynamics in I = 2, J = 2 wave.

As for comparison with results of Morgan and Pennington [7], it's difficult to say unambiguously does our S–wave contradict to their result or not. They have few solutions (with accounting of $\Gamma^{++}_{f_2\gamma\gamma}$ coupling or not), their preferable solution has resonance–like behavior of the I=0 S–wave cross section with much bigger value. This solution has the sizeable $\Gamma^{++}_{f_2\gamma\gamma}$ coupling. We here restricted ourselves by assumption $\Gamma^{++}_{f_2\gamma\gamma} = 0$, as it was made in [1]. We didn't meet serious contradictions with this assumption at least in the first approximation. The inclusion of this coupling into consideration needs the essential hypothesis because of interference effects.

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Table 1.

	1	2	3	4
	$\rho\ +\ \omega$, $s_{eff}=\infty$	$\rho\ +\ \omega$, $s_{eff}=1.69\ GeV^2$	All resonances, $s_{eff}=\infty$	All resonances, $s_{eff}=1.69\ GeV^2$
CELLO $\gamma\gamma\to\pi^+\pi^-$ $0.8\le E\le 1.5$	$R^0=0.290\pm0.009$ $(\alpha+\beta)^C=0.41\pm0.04$ $\chi^2/NDF=69/51$	0.291 ± 0.009 0.40 ± 0.08 $69.2/51$	0.290 ± 0.009 0.41 ± 0.04 $69/51$	0.291 ± 0.009 0.42 ± 0.08 $69/51$
MARK-II $\gamma\gamma\to\pi^+\pi^-$ $0.6\le E\le 1.5$	$R^0=0.282\pm0.014$ $(\alpha+\beta)^C=0.32\pm0.05$ $\chi^2/NDF=23.2/48$	0.286 ± 0.013 0.23 ± 0.09 $23.6/48$	0.283 ± 0.014 0.32 ± 0.05 $23.2/48$	0.285 ± 0.013 0.25 ± 0.09 $23.4/48$
CB $\gamma\gamma\to\pi^0\pi^0$ $0.85\le E\le 1.45$	$R^0=0.332\pm0.013$ $(\alpha+\beta)^N=1.24\pm0.06$ $\chi^2/NDF=44.0/47$	0.326 ± 0.013 1.56 ± 0.10 $42.0/47$	0.331 ± 0.013 1.32 ± 0.06 $43.6/47$	0.324 ± 0.013 1.62 ± 0.10 $41.6/47$

Table 2.

	Present work	Chiral models		Superconduct. quark model [14]	Quark–virton model [13]	Dispersion sum rules [15]
		One loop [22]	Two loops [16]			
$(\alpha+\beta)^C$	$0.41\pm0.08\pm0.01$ (CELLO) $0.28\pm0.09\pm0.05$ (MARK–II)	0	–	0.2	0.2	0.42 ± 0.05
$(\alpha+\beta)^N$	$1.43\pm0.10\pm0.20$ (Crystal Ball)	0	1.45 ± 0.38	1.20	1.2	1.61 ± 0.08

Tables captions:

- **Table 1** Best–fit D–wave parameters at fixed value $r = 5.5\,GeV^{-1}$, it corresponds to scattering length $a_2^0 = 1.7 \cdot 10^{-3}$. Different variants 1–4 correspond to different forms of background contribution (4). R^0 (the two–photon coupling constant of $f_2(1270)$) in units of GeV^{-2}, $\alpha + \beta$ in units of $10^{-42} cm^3$, $e^2 = 4\pi\alpha$.

- **Table 2** Comparison of obtained values for the sum of polarizabilities at $r = 5.5\ GeV^{-1}$ ($a_2^0 = 1.7 \cdot 10^{-3}$) with existing theoretical predictions.

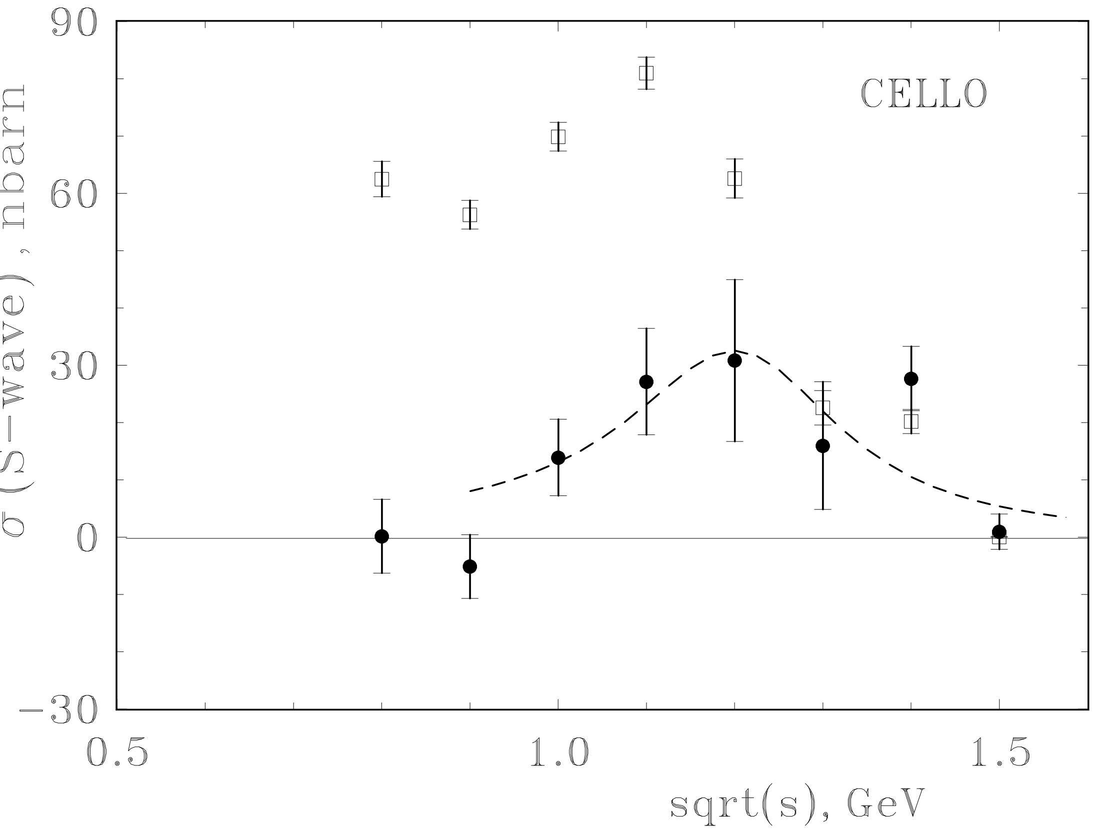
Figures captions:

- **Figure 1** Best–fit S–wave cross section $\mid cos\Theta \mid < 0.6$ from CELLO data [1] at fixed value $r = 5.5\ GeV^{-1}$. The points with central box are result of analysis [1] of the same data. For illustration there is shown the curve corresponding to scalar meson production with $M = 1200\ MeV,\ \Gamma = 300\ MeV$ and $\Gamma(\epsilon \to \gamma\gamma) \cdot BR(\epsilon \to \pi\pi) = 3.6\ KeV$.

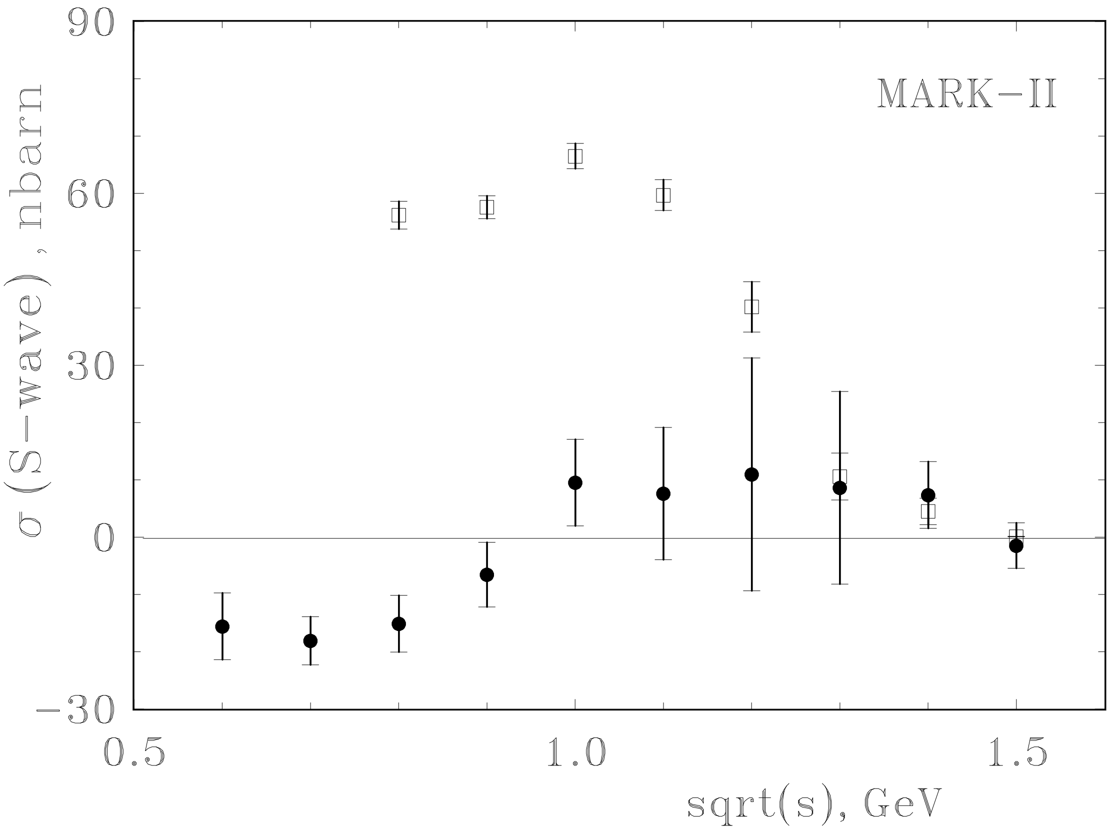
- **Figure 2** The same for MARK–II data [2].

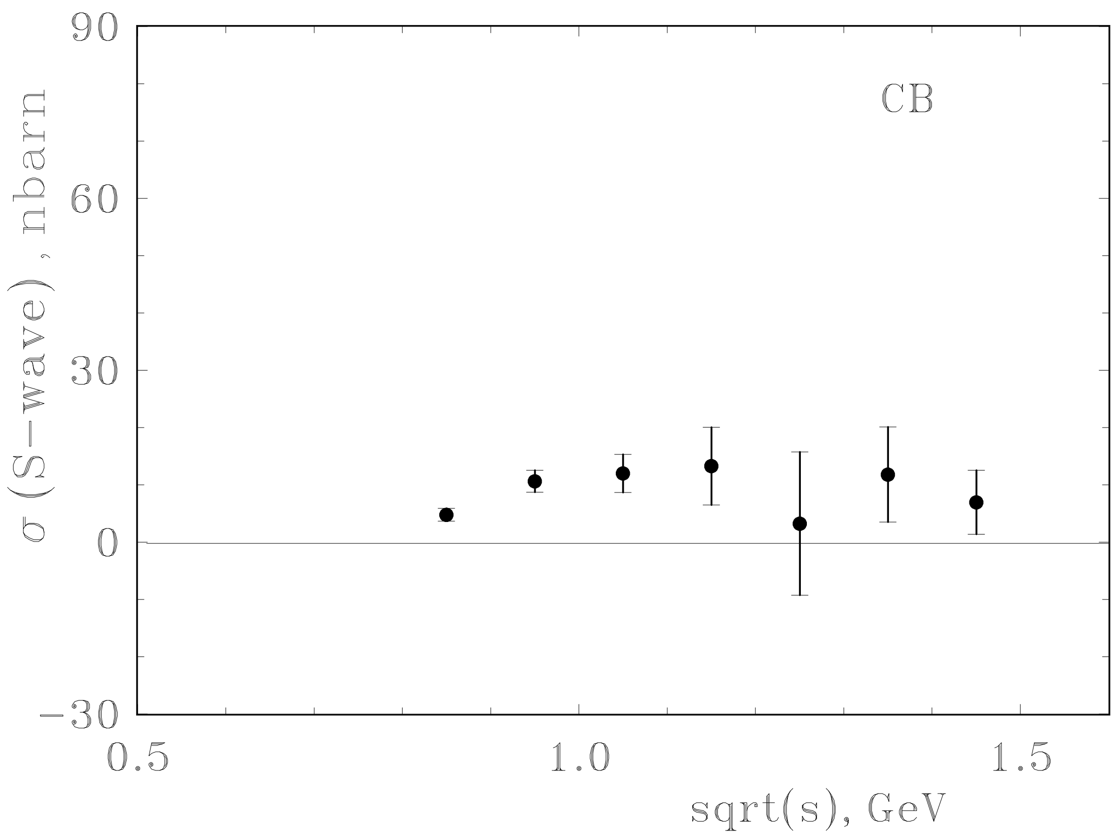
- **Figure 3** The same for Crystal Ball data [3], $\mid cos\Theta \mid < 0.7$.

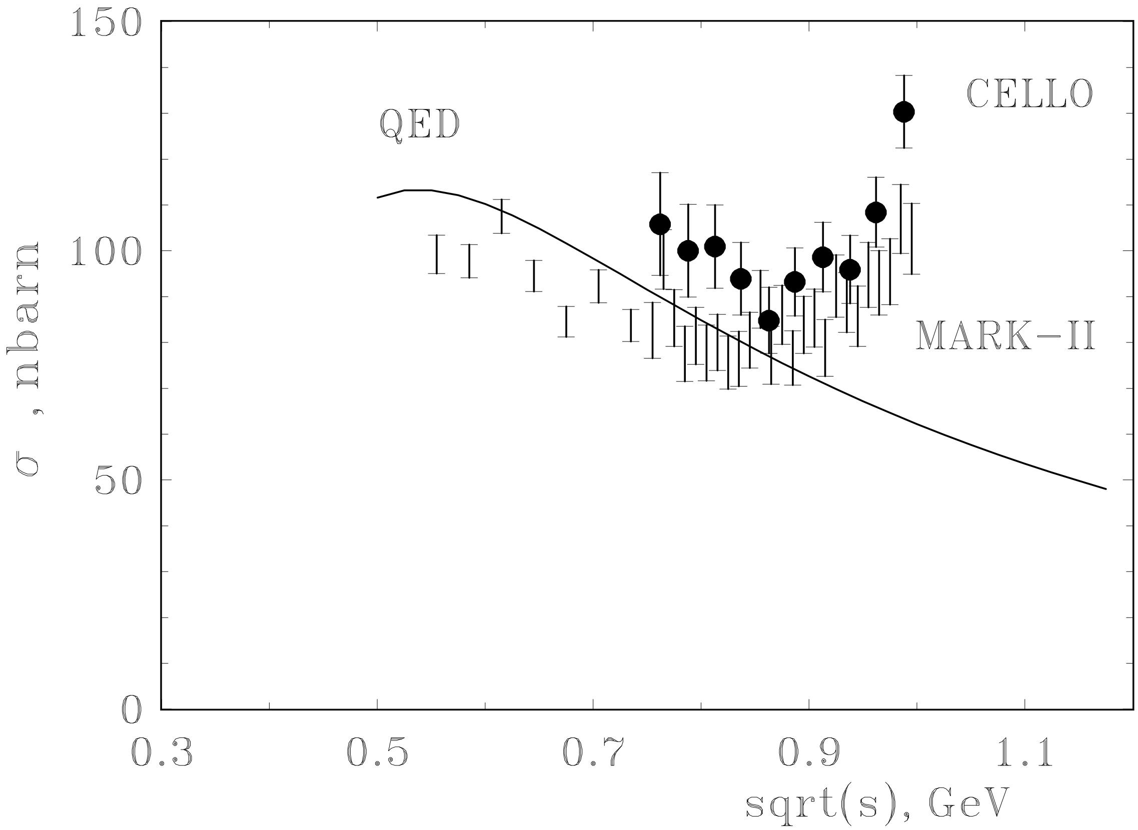
- **Figure 4** Integral cross sections of CELLO and MARK–II below 1 GeV, $\mid cos\Theta \mid < 0.6$, in comparison with π–exchange helicity 2 contribution (curve).



CELLO
σ (S−wave) , nbarn
sqrt(s), GeV
90
60
30
0
−30
0.5
1.0
1.5


90
60
30
0
−30
σ (S−wave) , nbarn
MARK−II
0.5
1.0
1.5
sqrt(s), GeV


CB
90
60
30
0
−30
0.5
1.0
1.5
σ (S−wave) , nbarn
sqrt(s), GeV



σ , nbarn
150
100
50
0
0.3
0.5
0.7
0.9
1.1
sqrt(s), GeV
QED
CELLO
MARK-II